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LAW OF REFRACTION AT A SPHERICAL REFRACTING SURFACE

Refraction Formula $\frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R}$ for a spherical surface separating two media by using Fermat's principle.

Suppose XY is a spherical refracting surface of pole P, centre of curvature C and radius of curvature R, which separates two media of refractive indices μ_1 and μ_2 ($\mu_2 > \mu_1$) as shown in figure-6.

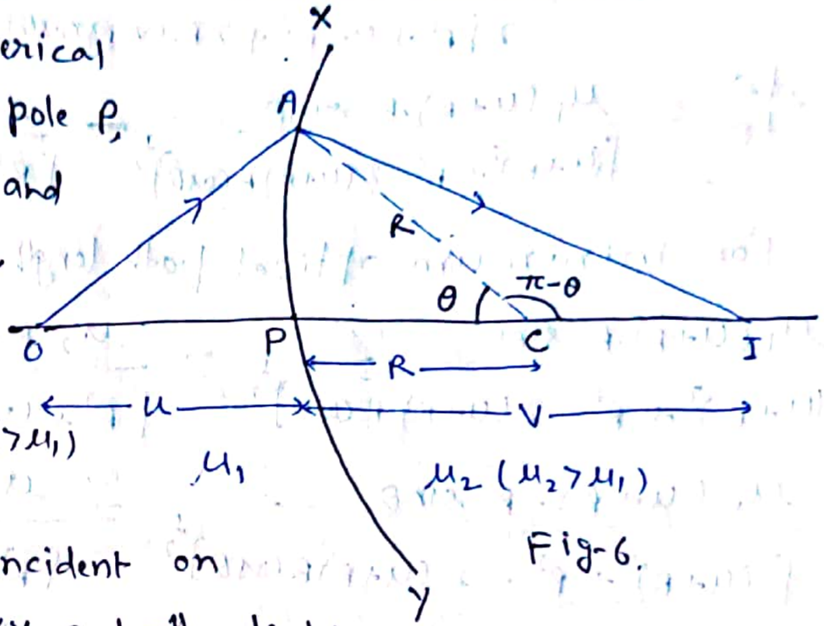

Fig-6.

Let a ray OA is incident on the spherical surface XY and the light ray is refracted by the spherical surface and after refraction, it reaches the point I.

Let $OP = u$ = object distance $PI = v$ = Image distance.

According to Fermat's principle, the optical path length connecting O and I must be minimum in comparision with all neighbouring paths of same general character.

The light ray OAI connecting the object point O and the image point has the optical path length

$$\Delta = \mu_1 \cdot OA + \mu_2 \cdot AI \quad \text{——— (1)}$$

Using cosine rule in $\triangle OAC$, $OA^2 = OC^2 + AC^2 - 2\,OC \cdot AC \cdot \cos\theta$

$$\Rightarrow OA = \left[(u+R)^2 + R^2 - 2\cdot(u+R)\cdot R\cdot\cos\theta\right]^{\frac{1}{2}} \quad \text{——— (2)}$$

Using cosine rule in $\triangle AIC$, $AI^2 = AC^2 + CI^2 - 2AC \cdot CI \cdot \cos(\pi-\theta)$

$$\Rightarrow AI = \left[R^2 + (v-R)^2 + 2\cdot R\cdot (v-R)\cdot\cos\theta\right]^{\frac{1}{2}} \quad \text{——— (3)} \quad \because \cos(\pi-\theta) = -\cos\theta$$

Now from eqn's (1), (2) and (3), optical path length Δ will be

$$\Delta = \mu_1\left[(u+R)^2 + R^2 - 2(u+R)R\cdot\cos\theta\right]^{\frac{1}{2}} + \mu_2\left[R^2 + (v-R)^2 + 2R(v-R)\cos\theta\right]^{\frac{1}{2}} \quad \text{——— (4)}$$

Diff. equⁿ (4) w. r. t. θ, we get

$$\frac{d\Delta}{d\theta} = \mu_1 \frac{1}{2\left[(u+R)^2 + R^2 - 2(u+R)\cdot R\cos\theta\right]^{\frac{1}{2}}} \times -2(u+R)R.(-\sin\theta)$$

$$+\mu_2 \cdot \frac{1}{2\left[R^2 + (v-R)^2 + 2R.(v-R)\cos\theta\right]^{\frac{1}{2}}} \times 2R.(v-R).(-\sin\theta)$$

$$\Rightarrow \frac{d\Delta}{d\theta} = \frac{\mu_1 (u+R).R.\sin\theta}{\left[(u+R)^2 + R^2 - 2(u+R)R\cos\theta\right]^{\frac{1}{2}}} - \frac{\mu_2 R(v-R).\sin\theta}{\left[R^2 + (v-R)^2 + 2R(v-R)\cos\theta\right]^{\frac{1}{2}}} \quad \text{--- (5)}$$

For minimum optical path length Δ, put $\frac{d\Delta}{d\theta} = 0$

$$\frac{\mu_1 (u+R).R\sin\theta}{\left[(u+R)^2 + R^2 - 2(u+R).R\cos\theta\right]^{\frac{1}{2}}} - \frac{\mu_2 R.(v-R)\sin\theta}{\left[R^2 + (v-R)^2 + 2R(v-R)\cos\theta\right]^{\frac{1}{2}}} = 0$$

$$\Rightarrow \frac{\mu_1 (u+R).R\sin\theta}{\left[(u+R)^2 + R^2 - 2(u+R)R\cos\theta\right]^{\frac{1}{2}}} = \frac{\mu_2 R(v-R)\sin\theta}{\left[R^2 + (v-R)^2 + 2R(v-R)\cos\theta\right]^{\frac{1}{2}}}$$

$$\Rightarrow \frac{\mu_1.(u+R)}{\left[(u+R)^2 + R^2 - 2(u+R)R\cos\theta\right]^{\frac{1}{2}}} = \frac{\mu_2.(v-R)}{\left[R^2 + (v-R)^2 + 2R(v-R)\cos\theta\right]^{\frac{1}{2}}}$$

when θ is small then cos θ = 1, using this we get

$$\frac{\mu_1 (u+R)}{\left[u^2 + R^2 + 2uR + R^2 - 2uR - 2R^2\right]^{\frac{1}{2}}} = \frac{\mu_2.(v-R)}{\left[R^2 + v^2 + R^2 - 2vR + 2vR - 2R^2\right]^{\frac{1}{2}}}$$

$$\Rightarrow \frac{\mu_1 (u+R)}{u} = \frac{\mu_2 (v-R)}{v}$$

$$\Rightarrow \mu_1 + \frac{\mu_1 R}{u} = \mu_2 - \frac{\mu_2 R}{v} \Rightarrow \frac{\mu_2 R}{v} + \frac{\mu_1 R}{u} = \mu_2 - \mu_1$$

$$\Rightarrow \frac{\mu_2}{v} + \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R} \quad \text{--- (6)}$$

Using sign convention, u = −u, v = +v and R = +R in equⁿ (6)

We get $\boxed{\frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R}}$ It is refraction formula for a spherical surface separating two media.